

No	SOLUTIONS TM																				
1.	<p>Using approximate formula to calculate YTM:</p> $\frac{k}{2} = \frac{\frac{C}{2} + \frac{(Bn - B_0)}{2n}}{\frac{(Bn + B_0)}{2}} = \frac{\frac{120}{2} + \frac{(1000 - 1100)}{20}}{\frac{(1000 + 1100)}{2}} = 5.24\%$ <p>Annualized yield = k = 5.24% x 2 = 10.48%</p> <p>[Using interpolation, we would get around 10.37%]</p>																				
2.	<p>Current yield = $\frac{C}{B_0}$</p> <p>Using the given data, we can write:</p> $8.21\% = \frac{80}{B_0}$ <p>Therefore, $B_0 = ₹974.42$</p> <p>Let us use interpolation to find implied YTM.</p> <table><tr><td></td><td></td><td>₹</td><td>₹</td></tr><tr><td>L</td><td>8.00%</td><td>1,000.00</td><td>1,000.00</td></tr><tr><td>K</td><td>UNKNOWN</td><td>₹ 974.42</td><td>₹ 0.00</td></tr><tr><td>H</td><td>10.00%</td><td>₹ 0.00</td><td>₹ 924.20</td></tr><tr><td></td><td></td><td>₹ 25.58</td><td>₹ 75.80</td></tr></table> <p style="text-align: center;">D1 D2</p>			₹	₹	L	8.00%	1,000.00	1,000.00	K	UNKNOWN	₹ 974.42	₹ 0.00	H	10.00%	₹ 0.00	₹ 924.20			₹ 25.58	₹ 75.80
		₹	₹																		
L	8.00%	1,000.00	1,000.00																		
K	UNKNOWN	₹ 974.42	₹ 0.00																		
H	10.00%	₹ 0.00	₹ 924.20																		
		₹ 25.58	₹ 75.80																		

$$YTM = k = L + \frac{D_1}{D_2}(H - L) = 8.67\% \text{ approx.}$$

3. $B_0 = C/2 \times PVIFA(k/2, 2n) + B_n \times PVIF(k/2, 2n)$
- $$1020 = C/2 \times PVIFA(10.5883\%/2, 2 \times 7) + 1000 \times PVIF(10.5883\%/2, 2 \times 7)$$
- Solving we get, $C = 11\%$
- Therefore, current yield $= \frac{C}{B_0} = \frac{110}{1020} = 10.78\%$

4. YTM can be calculated using interpolation:

L	8.00%	₹ 1,200.99	₹ 1,200.99
k	UNKNOWN	₹ 1,175.00	₹ 0.00
H	9.00%	₹ 0.00	₹ 1,127.76
		₹ 25.99	₹ 73.23
		D1	D2

$$YTM = k = L + \frac{D_1}{D_2}(H - L) = 8.35\% \text{ approx.}$$

YTC can be calculated either by approximate formula or by interpolation. Refer to problem solved in the class:

$$YTC = \frac{C + \frac{(B_n - B_0)}{n}}{\frac{(B_n + B_0)}{2}} = \frac{110 + \frac{(1000 + 90 - 1175)}{5}}{\frac{(1000 + 90 + 1175)}{2}} = 8.21\%$$

approx.

[Using interpolation, we would get answer closer to 8.13%]

5. Using YTM, we can find market price of the bond. Assuming that bond is fairly traded in the market, we can assume that market price is same as theoretical price. Therefore, using theoretical price formula we can find B_0 .

$$B_0 = C \times PVIFA(k\%, n) + B_n \times PVIF(k\%, n)$$

$$B_0 = 900 \times PVIFA(8.5\%, 15) + 1000 \times PVIF(8.5\%, 15)$$

$$= ₹1041.52$$

Then find B_5 , using the same parameters, with $k =$

10% and $n = 10$ years. We get:

$$B_0 = 90 \times \text{PVIFA}(10\%, 10) + 1000 \times \text{PVIF}(10\%, 10) \\ = ₹938.96$$

6. The current yield $= C/B_0 = ₹80/₹901.40 = 8.88\%$

YTM using approximate formula:

$$= \frac{C + \frac{(Bn - B_0)}{n}}{\frac{(Bn + B_0)}{2}} = \frac{80 + \frac{(1000 - 901.40)}{9}}{\frac{(1000 + 901.40)}{2}} = 9.6\% \text{ approx.}$$

[Using interpolation, we would get answer closer to 9.7%]

After 1 year, assuming the YTM to be same at 9.7%, the bond price:

$$B_1 = 80 \times \text{PVIFA}(9.7\%, 8) + 1000 \times \text{PVIF}(9.7\%, 8) \\ = ₹908.52$$

Therefore, Capital gain = ₹908.52 - ₹901.40 = ₹7.12

[The correct answer is ₹7.12]

37. [Problem number incorrectly printed as 37...Assume it as 7a]

The issue price of the debentures will be the sum of present value of interest payments during 10 years of its maturity and present value of redemption value of debenture.

Years	Cash out flow (~)	PVIF @ 16%	PV
1	9	0.862	7.76
2	9	0.743	6.69
3	9	0.641	5.77
4	9	0.552	4.97
5	10	0.476	4.76
6	10	0.41	4.10
7	10	0.354	3.54
8	10	0.305	3.05
9	14	0.263	3.68
10	14 + 100 + 5	0.227	27.01
Present Value of all Future Cash Flows =			₹71.33

The issue price of debenture would be ~71.33

7. Price of the semi-annual bond is ₹933.02.

$$B_0 = C \times PVIFA (k\%/2, 2n) + B_n \times PVIF (k\%/2, 2n)$$

$$PVIFA (4\%, 8) = 6.734 \text{ and } PVIF (4\%, 8) = 0.731$$

$$933.02 = C/2 \times 6.734 + 1000 \times 0.731$$

Solving, we get $C/2 = ₹30$ or 3% or $C = 6\%$.

8. Note that both the bonds have same tenure. Hence, they are expected to quote at same yield. Therefore, we find the yield of Bond A as (same as coupon, since it quotes at par) = 8%

Using $k = 8\%$, the price of Bond B will be:

$$B_0 = 140 \times PVIFA (8\%/2, 3 \times 2) + 1000 \times PVIF$$

$$(8\%/2, 2 \times 3)$$

$$= ₹1157.26$$

Year	Period	Coupons	PVF	PV of CF	Year x PV of CF
1	1	70	0.962	67.31	67.31
1	2	70	0.925	64.72	129.44
2	3	70	0.889	62.23	186.69
2	4	70	0.855	59.84	239.35
3	5	70	0.822	57.53	287.67
3	6	1070	0.790	845.63	5073.82

				Price	1157.26	5984.28
	Duration = ₹5984.28/₹1157.26 = 5.17 half-years					
	D = 5.17/2 = 2.59 years approx.					
9.	<p>Use duration formula we can calculate required answers of 3.53 years and 4 years respectively.</p> <p>The percentage change in price of Bond 1, if the interest rates rise by 2%, is found out as follows:</p> <p>Δ (i) = +2%, so that:</p> $\% B_0 = -D \times [\Delta (i) / 1 + i]$ <p>Bond 1:</p> $\% B_0 = -3.53 \times [+2/(1+0.09)]\% = -6.48\%$ <p>Bond 2:</p> $\% B_0 = -4 \times [+2/(1+0.09)]\% = -7.34\%$					
10.	<p>For semi-annual bond, the formula is:</p> $MD = D/(1 + k/2) = 6.72/(1 + 0.125/2) = 6.32 \text{ years}$					
11.	<p>The following points will be kept in mind before ranking:</p>					

- a) Longest tenure bonds will have larger durations.
- b) All other parameters being equal, between ZCB and coupon bond, ZCB's have higher duration.
- c) Lower the coupon, higher the duration.
- d) Higher the yield, lower the duration.

This explains, why 20-year bond have larger duration than 15-year bonds. C being ZCB, will have highest Duration. Then lower coupon bond D and next will be higher coupon bond A.

Bonds with higher yield have lower duration. This explains the order of B and E. Therefore, we have C, D, A, B and E, as the correct descending order.

12. The calculation is as under:

Di	3.861	8.047	9.168	
Prices	100.00	84.65	137.90	322.55
Wi	0.3100	0.2624	0.4276	1.0000
Wi x Di	1.20	2.11	3.92	7.23

Duration of bond portfolio = 7.23 years

13.

The two-year asset will yield same income in both years, since the rate (10%) is decided today. Against this 2-year asset, we have 1-year liability, which costs 8% in 1st year and 9% for the 2nd year. Therefore, the liability is not same in both years. Therefore, net interest income will decrease in the second year.

	<u>Year 1</u>	<u>Year 2</u>
Interest income	₹5,000,000	₹5,000,000
Interest expense	₹4,000,000	₹4,500,000
Net interest income	₹1,000,000	₹500,000

14. i)

Year	CF	k = 8%	PV of CF	w x PVCF	Duration
1	100	0.926	92.6	92.6	
2	1100	0.857	942.7	1885.4	
			1035.3	1978	1.911

Year	CF	k = 10%	PV of CF	w x PVCF	Duration
1	100	0.909	90.9	90.9	
2	1100	0.826	908.6	1817.2	

			999.5	1908.1	1.909
Year	CF	k = 12%	PV of CF	w x PVCF	Duration
1	100	0.893	89.3	89.3	
2	1100	0.797	876.7	1753.4	
			966	1842.7	1.908

ii) Duration of ZCB = n. Therefore, irrespective of different YTM, D = 2 years for all values of YTM.

iii) Answer given

15. Use duration formula and calculate the duration for the 3 different yields (k).

16. Use duration formula and calculate the duration for the 3 different maturities (n).

17. We can use the duration formula to calculate the required durations. Alternatively:

Duration calculation:

Year	C or C + Bn	PV @6%	PVCF	Wt x PV of CF
1	600	0.943	566.04	566.0377
2	600	0.890	534.00	1067.9957
3	600	0.840	503.77	1511.3147
4	600	0.792	475.26	1901.0248
5	10600	0.747	7920.94	47077.26
TOTAL			10000.00	52123.6379

Duration calculation (semi-annual):

Year	C or C	PV @3%	PVCF	Wt x PV of
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	+ Bn			CF
1	300	0.971	291.26	291.2621
1	300	0.943	282.78	565.5575
2	300	0.915	274.54	823.6275
2	300	0.888	266.55	1066.1845
3	300	0.863	258.78	1293.9132
3	300	0.837	251.25	1507.4717
4	300	0.813	243.93	1707.4922
4	300	0.789	236.82	1894.5822
5	300	0.766	229.93	2069.3252
5	300	0.744	223.23	2232.2817
6	300	0.722	216.73	2383.9902
6	10300	0.701	7224.21	86690.5532
			10000.00	102526.24

$D = 102526.24/10000 = 10.2526$ half-years
 Therefore, $D = 10.2526/2 = 5.126$ years

18. We can use the duration formula to calculate the required durations.

19. We can use the duration formula to calculate the required duration.

20.
$$MD = \frac{D}{1 + \frac{k}{2}} = \frac{6.763}{1 + \frac{0.10}{2}} = 6.441 \text{ years}$$

21. We can use the duration formula to calculate the required duration.

22. Refer class notes – solution for problem 24.
 i.

Year	CF	k = 9%	PV of CF	w x PVCF	Duration
1	150	0.917	137.55	137.55	
2	150	0.842	126.3	252.6	
3	150	0.772	115.8	347.4	

4	150	0.708	106.2	424.8	
5	1150	0.650	747.5	3737.5	
			1233.35	4899.85	3.97

Duration = 4 years approx.

ii. [Reinvestment rate at 10%]

At the end of 4 years, we receive the following:

Coupons = $4 \times 150 = ₹600$

Re-investment income =

$150 \times 1.1^3 + 150 \times 1.1^2 + 150 \times 1.1^1 - 3 \times 150 = ₹96.15$

Bond Price at the end of year 4 = $B_4 = 1150/1.1 = ₹1045.45$

Total income = $₹600 + ₹96.15 + ₹1045.45 = ₹1741.60$

Realized yield can be calculated using the formula:

$$B_0 = \frac{\text{Total Cash Flow}}{(1 + R)^4}$$

$$1233.35 = \frac{1741.60}{(1 + R)^4}$$

$$\frac{1}{(1 + R)^4} = 0.708$$

From PVIF table, we see that $R = 9\%$

iii. [Reinvestment rate at 8%]

At the end of 4 years, we receive the following:

Coupons = $4 \times 150 = ₹600$

Re-investment income =

$150 \times 1.08^3 + 150 \times 1.08^2 + 150 \times 1.08^1 - 3 \times 150 = ₹75.92$

Bond Price at the end of year 4 = $B_4 = 1150/1.08 = ₹1064.81$

Total income = $₹600 + ₹75.92 + ₹1064.81 = ₹1740.73$

Realized yield can be calculated using the formula:

$$B_0 = \frac{\text{Total Cash Flow}}{(1 + R)^4}$$

$$1233.35 = \frac{1740.73}{(1 + R)^4}$$

Again, we get,

$$\frac{1}{(1 + R)^4} = 0.708$$

From PVIF table, we see that $R = 9\%$

iv. Irrespective of different YTM, the realized yield is same. This happens because our investment period (4 years) matches with the duration of the bond (4 years approx.). Whenever, we match investment period with duration of bond investment, the rise / fall in reinvestment income offsets fall / rise in sale price of bond.

23. Duration of asset = duration of ZCB = $n = 7$ years.

Liability details:

$n = 10$ years, $C = 8.275\%$, $B_n = 10,00,000$, $k = 10\%$

Duration is as calculated below:

Year	C or C + Bn	PV Factor	Price Today	Wt x PV of CF
1	82750	0.909	75227.27	75227.2727
2	82750	0.826	68388.43	136776.8595
3	82750	0.751	62171.30	186513.8993
4	82750	0.683	56519.36	226077.4537
5	82750	0.621	51381.24	256906.1974
6	82750	0.564	46710.22	280261.3063
7	82750	0.513	42463.83	297246.8400
8	82750	0.467	38603.49	308827.8857
9	82750	0.424	35094.08	315846.7013
10	1082750	0.386	417447.00	4174469.9663

TOTAL

894006.22 6258154.3822

$$D = 6258154.38 / 8946006.22 = 7 \text{ years}$$

24.

i. Duration calculation:

1	30	0.971	29.13	29.1262
2	30	0.943	28.28	56.5558
3	30	0.915	27.45	82.3627
4	30	0.888	26.65	106.6184
5	30	0.863	25.88	129.3913
6	30	0.837	25.12	150.7472
7	30	0.813	24.39	170.7492
8	30	0.789	23.68	189.4582
9	30	0.766	22.99	206.9325
10	1030	0.744	766.42	7664.1673
TOTAL			1000.00	8786.1089

$$D = 8786.11 / 1000 = 8.79 \text{ half-years}$$

$$\text{Therefore, } D = 8.79 / 2 = 4.39 \text{ years}$$

ii. D_A calculation given below:

Value	Wi	Di	WiDi
90	0.0296	0.5	0.015
55	0.0181	0.9	0.016
176	0.0578	4.39	0.254
2724	0.8946	7	6.262
3045	1.0000		6.547

iii. D_L calculation given below:

Value	Wi	Di	WiDi
2092	0.8979	1.000	0.8979
238	0.1021	0.010	0.0010
2330	1.0000		0.8989

25.

Narration	Benefits	Narration	Cost
PV of Interest Savings	₹ 31,44,000.00	Net Cost of Premium paid	₹ 7,50,000.00
Amort. Of Fl Cost (New)	₹ 3,14,400.00	Fl. Cost of New Bond	₹ 9,00,000.00
Net benefit from Amort of Fl. Cost (Old)	₹ 54,240.00		
Total Benefits	₹ 35,12,640.00	Total Costs	₹ 16,50,000.00
Net Benefit	₹ 18,62,640.00		

26.

Narration	Benefits	Narration	Cost
PV of Interest Savings	₹ 38,43,000.00	Net Cost of Premium paid	₹ 25,20,000.00
Amort. Of Fl Cost (New)	₹ 68,320.00	Fl. Cost of New Bond	₹ 4,00,000.00
Net benefit from Amort of Fl. Cost (Old)	₹ 68,760.00	Overlap Interest	₹ 4,20,000.00
Net benefit from Amort of discount	₹ 1,71,900.00		

(Old)			
Total Benefits	₹ 41,51,980.00	Total Costs	₹ 33,40,000.00
Net Benefit	₹ 8,11,980.00		

27.

Narration	Benefits	Narration	Cost
PV of Interest Savings	₹ 74,77,200.00	Net Cost of Premium paid	₹ 36,00,000.00
Amort. Of Fl Cost (New)	₹ 62,310.00	Fl. Cost of New Bond	₹ 2,50,000.00
Net benefit from Amort of Fl. Cost (Old)	₹ 24,121.60	Overlap Interest	₹ 6,00,000.00
Net benefit from Amort of discount (Old)	₹ 1,20,608.00		
Total Benefits	₹ 76,84,239.60	Total Costs	₹ 44,50,000.00
Net Benefit	₹ 32,34,239.60		

28.

Year	1	2	3
r	r_1	8%	r_3
k	6%	k_2	9%

We want to find r_3 .

We know that $r_1 = k_1$. Therefore, $r_1 = 6\%$.

Using r_1 , k_1 and r_2 , we can find k_2 .

We use the formula:

$$1 + r_n = \frac{(1 + k_n)^n}{(1 + k_{n-1})^{n-1}}$$

$$1 + r_2 = \frac{(1 + k_2)^2}{(1 + k_1)^1}$$

Substituting and solving, we get $k_2 = 7\%$ approx.

Again, using the same formula, we can write,

$$1 + r_3 = \frac{(1 + k_3)^3}{(1 + k_2)^2}$$

Substituting and solving, we get $r_3 = 13.1\%$

29.

We have:

Year	1	2
r	r_1	6.5%
k	5%	k_2

We want to find k_2 .

We know that $r_1 = k_1$. Therefore, $r_1 = 5\%$.

Using r_1 , k_1 and r_2 , we can find k_2 .

We use the formula:

$$1 + r_n = \frac{(1 + k_n)^n}{(1 + k_{n-1})^{n-1}}$$

$$1 + r_2 = \frac{(1 + k_2)^2}{(1 + k_1)^1}$$

Substituting and solving, we get $k_2 = 5.75\%$ approx.

30.

We have:

Year	1	2	3
r	r_1	6.5%	8%
k	5%	k_2	k_3

We want to find k_3 .

We know that $r_1 = k_1$. Therefore, $r_1 = 5\%$.

Using r_1 , k_1 and r_2 , we can find k_2 .

We use the formula:

$$1+r_n = \frac{(1+k_n)^n}{(1+k_{n-1})^{n-1}}$$

$$1+r_2 = \frac{(1+k_2)^2}{(1+k_1)^1}$$

Substituting and solving, we get $k_2 = 5.75\%$ approx.

Again, using the same formula, we can write,

$$1+r_3 = \frac{(1+k_3)^3}{(1+k_2)^2}$$

Substituting and solving, we get $k_3 = 6.49\%$ approx.

31. We have:

Year	1	2	3
r	r_1	r_2	r_3
k	5%	7%	10%

We want to find r_2 and r_3 .

We know that $r_1 = k_1$. Therefore, $r_1 = 5\%$.

Using the formula given in previous problem, we have:

$$r_2 = \frac{(1+k_2)^2}{(1+k_1)^1} - 1 \text{ and } r_3 = \frac{(1+k_3)^3}{(1+k_2)^2} - 1$$

Substituting the given values, we get:

$$r_2 = \frac{(1.07)^2}{(1.05)^1} - 1 \text{ and } r_3 = \frac{(1.1)^3}{(1.07)^2} - 1,$$

Therefore, by solving, we get $r_2 = 9.04\%$ and $r_3 = 16.25\%$ respectively.

32. a) Conversion Value of the bond = Conversion ratio x Current market price

$$= 25 \times \sim 30 = \sim 750$$

b) Currently the convertible is available at ~ 1000 ; on conversion we would get 25 shares.
Market conversion price = $\text{₹}1000/25 = \text{₹}40$

c) Implied conversion price on buying the convertible today = ~ 40 (as calculated above).
This is ~ 10 more than the current market price of equity share i.e. ~ 30 . Conversion premium per share = Implied conversion price – current market price = $\text{₹}40 - \text{₹}30 = \text{₹}10$

d) Ratio of conversion premium = conversion premium calculated as % :

$$\frac{40-30}{30} \times 100 = 33.33\%$$

e) Straight value of the bond = $\text{₹}800$ (given)

Market price of convertible = $\text{₹}1000$

Premium over straight value = $\text{₹}1000 - \text{₹}800 = \text{₹}200$

In % terms it is: $\frac{1000-800}{800} \times 100 = 25\%$

f) In this problem the investor is buying the convertible at the current market price of ~ 1000 . Since the conversion ratio is 25:1, he will receive 25 shares. Now if the equity shares give ~ 1 dividend per share, we can say that on conversion to 25 shares, his dividend income is ~ 25 per annum. The coupon from the underlying bond before conversion per annum is ~ 90 (9% of ~ 1000). The income difference is $\sim 90 - \sim 25 = \sim 65$ and per share it is $\sim 65 / 25 = \sim 2.60$ per share. This favourable income per

share by not converting is ₹2.60 per share per annum.

- g) If the investor purchases the convertible from the market, he will pay ₹10 per share as premium. By holding the bond till conversion his incremental income is ₹2.60 per share per annum. Thus ₹10 will be paid back at the rate of ₹2.60 per annum. Payback period is therefore, $\text{₹}10/\text{₹}2.60 = 3.85$ years

AJ EDUCATION NEXT